1.2 Recognising Perfect Squares

GOAL

Use a variety of strategies to identify perfect squares.

?

1. Use a tree diagram to express each number as the product of two identical numbers.

   a) 400

   c) 81

   b) 196

   d) 144

At-Home *Help*

The product of two identical numbers is a perfect square.

You can use a tree diagram to determine if a number is a perfect square.

For example, is 225 a perfect square?

225
^
5 45
^
5 9
^
3 3

$225 = 5 \times 5 \times 3 \times 3$

$= (5 \times 3) \times (5 \times 3)$

$= 15 \times 15$

Yes, 225 is a perfect square.

2. Calculate.

   a) 10^2 b) 16^2 c) 18^2 d) 21^2

3. Express each number as a perfect square and write the final answer in the form $\blacksquare^2$. Use a tree diagram to determine the factors.

   a) 4900 b) 324 c) 625 d) 576

4900
^
10 490
^

1.3 Square Roots of Perfect Squares

GOAL

Use a variety of strategies to determine the square root of a perfect square.

1. Calculate.

   a) $\sqrt{9}$

   b) $\sqrt{900}$

   c) $\sqrt{196}$

   d) $\sqrt{625}$

2. A bulletin board has an area of 1296 cm^2.

   a) Determine three possible sets of lengths and widths for the bulletin board. Include a diagram for each set.

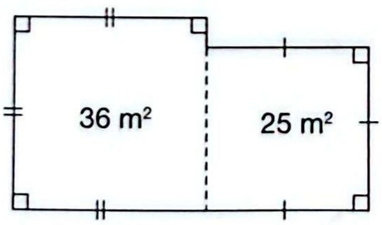
   ____ m × ____ m   ____ m × ____ m   ____ m × ____ m

   b) Which length and width forms a square? ____ m × ____ m

   c) How else can you determine the possible length and width of the corkboard?

   ____

3. Consider the following diagram of two rooms. What is the total perimeter of the figure?



At-Home Help

The square root of a number is one of its two identical factors.

For example, the square root of 100 is 10 since 10 × 10 = 100.

You can represent a square root using the symbol $\sqrt{\ }$, under which the entire number is placed.

For example, $\sqrt{121} = 11$, and is read "the square root of 121 equals 11."

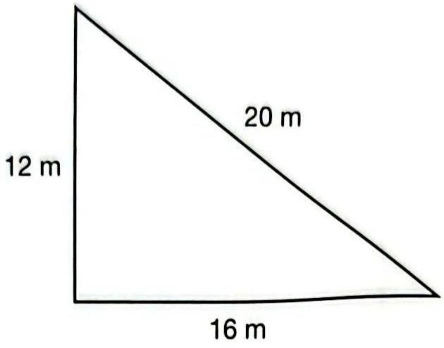
1.6 The Pythagorean Theorem

GOAL

Model, explain, and apply the Pythagorean theorem.

?

1. Is this triangle a right triangle?



$c^2 = a^2 + b^2$

$20^2 = 12^2 + 16^2$

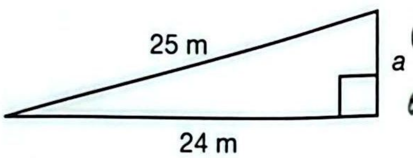
$400 = 144 + 256$

$400 = 400$

RS = LS? yes

Is this triangle a right triangle? yes.

2. Calculate the unknown length. $c^2 = a^2 + b^2$



$(25)^2 = a^2 + (24)^2$

$625 = a^2 + 576$

$a = \sqrt{49} = \boxed{7\text{ m}}$

3. Which of these are Pythagorean triples?

a) 11, 22, 30 ✗ **b)** 5, 12, 13 ✓

$(30)^2 \neq (11)^2 + (22)^2$

$(13)^2 = (5)^2 + (12)^2$

$169 = 25 + 144$

4. The hypotenuse of an isosceles right triangle is 14 cm. How long are the legs? Include a diagram. Recall that an isosceles right triangle has legs that are the same length.

$c^2 = a^2 + a^2$

$(14)^2 = 2(a^2)$

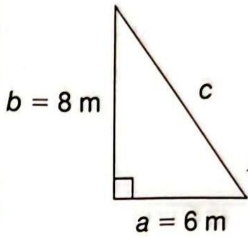
$\frac{196}{2} = a^2$

$a = \sqrt{98} \approx 9.9\text{ cm}$

At-Home Help

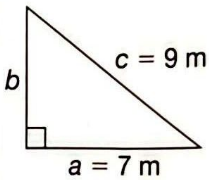
You can use the Pythagorean theorem to:

- calculate the hypotenuse



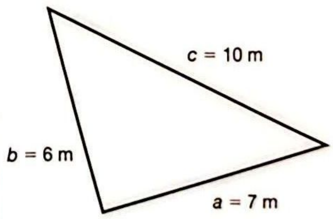
$c^2 = a^2 + b^2$
$c^2 = 6^2 + 8^2$
$c^2 = 36 + 64$
$c^2 = 100$
$\sqrt{c^2} = \sqrt{100}$
$c = 10\text{ m}$

- calculate one leg



$b^2 = c^2 - a^2$
$b^2 = 9^2 - 7^2$
$b^2 = 81 - 49$
$b^2 = 32$
$\sqrt{b^2} = \sqrt{32}$
$b \approx 6\text{ m}$

- determine if a triangle is a right triangle



$a^2 + b^2 = 7^2 + 6^2$
$= 49 + 36$
$= 85$
$c^2 = 10^2$
$= 100$
$85 \neq 100$

This triangle is not a right triangle.

1.7 Solve Problems Using Diagrams

GOAL

Use diagrams to solve problems about squares and square roots.

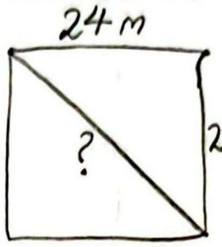
Draw a diagram to help you solve each problem.

1. A baseball diamond is a square with sides of about 24 m. What is the shortest distance, to the nearest tenth, between first base and third base?

$c^2 = (24)^2 + (24)^2$

$\sqrt{c^2} = \sqrt{1152}$

$c \approx 33.9$ m

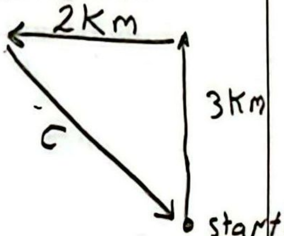


2. Two joggers ran 3 km north, then 2 km west. What is the shortest distance they must travel to return to their starting point?

$c^2 = (2)^2 + (3)^2$

$= 4 + 9$

$c = \sqrt{13} \approx 3.6$ m.

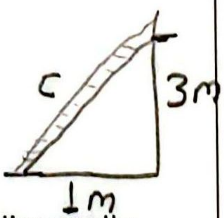


3. The foot of a ladder is placed 1 m away from a wall. The top of the ladder rests 3 m up the wall. How long is the ladder? $c^2 = a^2 + b^2$

$= (1)^2 + (3)^2$

$c^2 = 10$

$c = \sqrt{10} \approx 3.16$ m



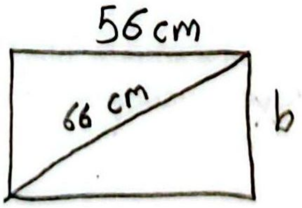
4. Daisy's TV screen is 56 cm long and 66 cm diagonally. How wide is Daisy's TV?

$c^2 = a^2 + b^2$

$(66)^2 = (56)^2 + b^2$

$4356 = 3136 + b^2$

$\sqrt{b^2} = \sqrt{1220}$ ⇒ $b = 34.9$ m



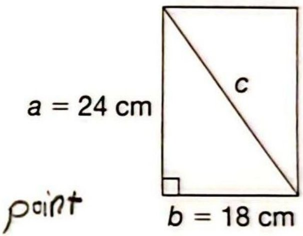
At-Home Help

A piece of wood is 24 cm long and 18 cm wide. Ian wants to cut the wood diagonally to make two shelf supports. What is the length of the diagonal?

When solving word problems in math, the following steps will help you:

1. Understand the problem.

   Draw a diagram of the situation.



2. Make a plan.

   Use the Pythagorean theorem to calculate the length of the diagonal.

3. Carry out the plan.

$$c^2 = a^2 + b^2$$
$$c^2 = 24^2 + 18^2$$
$$c^2 = 576 + 324$$
$$c^2 = 900$$
$$\sqrt{c^2} = \sqrt{900}$$
$$c = 30 \text{ cm}$$

The length of the diagonal for each shelf support is 30 cm.

2.1 Multiplying a Whole Number by a Fraction

GOAL

Use repeated addition to multiply fractions by whole numbers.

1. **a)** Write $4 \times \frac{3}{5}$ as a repeated addition sentence.

   ____ + ____ + ____ + ____ = ____

   b) Draw a model to calculate the answer.

2. Multiply. Write your answer as a fraction and as a mixed number.

   a) $7 \times \frac{2}{5} =$ ____________ or ____________

   b) $4 \times \frac{3}{8} =$ ____________ or ____________

3. How much farther are 3 jumps of $\frac{2}{5}$ on a number line than 2 jumps of $\frac{3}{5}$? Explain.

   ____

4. Math class is $\frac{4}{6}$ h for three days of each school week. How many hours of math class does a student have in one week?

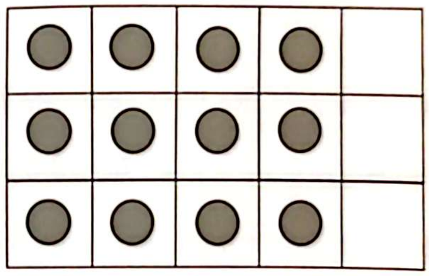
   ____ × ____ = ____

At-Home | *Help*

To multiply a whole number by a fraction, you can:

Multiply using grids and counters.

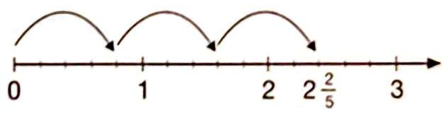
$3 \times \frac{4}{5}$ means the same as 3 sets of $\frac{4}{5}$.



Each square represents $\frac{1}{5}$, so the 12 squares covered with counters represent $\frac{12}{5}$.

Multiply using a number line.

Calculate $3 \times \frac{4}{5}$ using a number line. Write the product both as an improper fraction and as a whole or mixed number.



$$3 \times \frac{4}{5} = 3 \times 4 \text{ fifths}$$
$$= 12 \text{ fifths}$$
$$= \frac{12}{5} \text{ or } 2\frac{2}{5}$$

2.3 Multiplying Fractions

GOAL

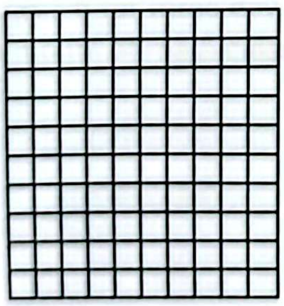
Multiply two fractions less than 1.

1. Draw a fraction strip model to determine the product of $\frac{4}{5} \times \frac{1}{2}$.

2. Match each expression with its product in the box to the right.

a) $\frac{3}{4} \times \frac{2}{5}$	i) $\frac{2}{5}$
b) $\frac{3}{5} \times \frac{2}{3}$	ii) $\frac{1}{15}$
c) $\frac{1}{6} \times \frac{2}{5}$	iii) $\frac{3}{10}$
d) $\frac{3}{8} \times \frac{4}{9}$	iv) $\frac{1}{6}$

3. a) Shade the grid to show why $\frac{2}{3} \times \frac{5}{8} = \frac{10}{24}$.



b) List two other pairs of fractions with a product of $\frac{10}{24}$.

____ × ____ ____ × ____

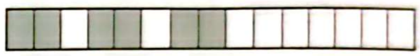
At-Home | Help

To multiply $\frac{2}{3} \times \frac{3}{5}$, you can:

Use a fraction strip model.

Model $\frac{3}{5}$ and divide each fifth into thirds.

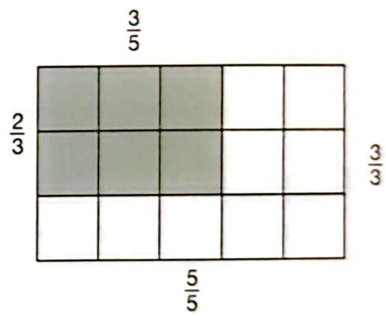
Then, to show $\frac{2}{3}$ of each section, colour 2 of the thirds.



$\frac{2}{3} \times \frac{3}{5} = \frac{6}{15}$

Use a grid model.

Calculate the area of a rectangle $\frac{2}{5}$ of a unit wide and $\frac{2}{3}$ of a unit long.



The coloured rectangle has $2 \times 3 = 6$ squares, so its area is $\frac{6}{15}$ square units.

$\frac{2}{3} \times \frac{3}{5} = \frac{6}{15}$

Multiply.

You can multiply the numerators and the denominators. For example,

$\frac{2}{3} \times \frac{3}{5} = \frac{2 \times 3}{3 \times 5}$ or $\frac{6}{15}$

2.5 Multiplying Fractions Greater than 1

GOAL

Multiply mixed numbers and improper fractions.

1. Estimate each product.

a) $1\frac{1}{3} \times 1\frac{2}{3}$ is close to ____ $\times$ ____ $=$ ____

b) $3\frac{1}{5} \times 6\frac{3}{8}$ is close to ____ $\times$ ____ $=$ ____

2. Multiply the following using an area model.

a) $1\frac{1}{2} \times 1\frac{1}{3} =$ ____

b) $3\frac{1}{5} \times 2\frac{1}{4} =$ ____

3. Multiply using improper fractions.

a) $2\frac{1}{5} \times 3\frac{1}{6} =$ ____ $\times$ ____ or ____

b) $2\frac{1}{4} \times 3\frac{1}{3} =$ ____ $\times$ ____ or ____

4. The product of three improper fractions is $\frac{5}{2}$. What could the three fractions be?

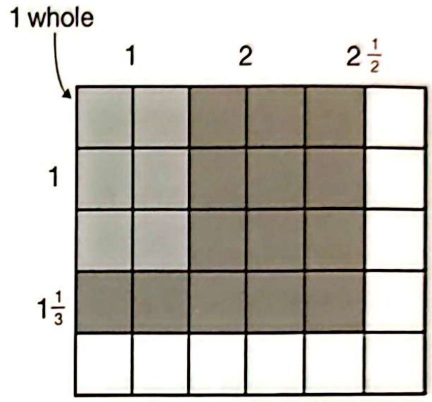
$\frac{5}{2} =$ ____ $\times$ ____ $\times$ ____

At-Home Help

To multiply mixed fractions, you can:

Use an area model.

For example, to calculate the area of a rectangle $1\frac{1}{3}$ units long and $2\frac{1}{2}$ units wide:



$1\frac{1}{3} \times 2\frac{1}{2} = \frac{20}{6}$ or $\frac{4}{3} \times \frac{5}{2} = \frac{20}{6}$

There are 6 squares in a whole, so each square is $\frac{1}{6}$. There are 20 squares in the coloured rectangle.

Another way to record this is $1 + 1 + \frac{8}{6} = 2\frac{8}{6}$, or $3\frac{2}{6}$, or $3\frac{1}{3}$

There are two wholes and 8 other squares in the coloured rectangle.

Multiply improper fractions.

Write each mixed number as an improper fraction and multiply as you would with proper fractions.

$$1\frac{1}{3} \times 2\frac{1}{2} = \frac{4}{3} \times \frac{5}{2} = \frac{4 \times 5}{3 \times 2} = \frac{20}{6}$$

2.6 Dividing Fractions by Whole Numbers

GOAL

Use a sharing model to represent the quotient of a fraction divided by a whole number.

1. Use a model to show $\frac{6}{9} \div 4 = \frac{1}{6}$.

2. a) How can you solve $\frac{2}{3} \div 5$ using multiplication of fractions?

   $\_\_\_\_ \times \frac{2}{3} = \_\_\_\_$

   b) Explain why this works.

   ____

   ____

3. Divide.

   a) $\frac{5}{3} \div 10 = \_\_\_\_$

   $= \_\_\_\_$

   b) $\frac{2}{7} \div 4 = \_\_\_\_$

   $= \_\_\_\_$

4. $\frac{4}{5}$ of a room has to be painted. 3 painters are going to share the job. What fraction of the room will each painter complete if they all paint at the same rate?

At-Home | *Help*

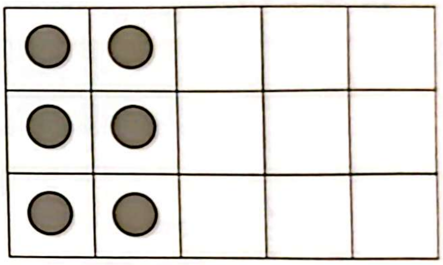
To divide a fraction by a whole number, you can:

Think of it as sharing.

For example, $\frac{2}{5} \div 3$ tells you the share size if 3 people share $\frac{2}{5}$ of something.

You cannot share $\frac{2}{5}$ equally among three people, so write an equivalent fraction.

$\frac{2}{5} = \frac{6}{15}$



$\frac{3}{15}$ are in each group,
so $\frac{2}{5} \div 3 = \frac{2}{15}$.

Multiply by a fraction.

For example, $\frac{2}{5} \div 3$ is the same as $\frac{1}{3}$ of $\frac{2}{5}$, or $\frac{1}{3} \times \frac{2}{5} = \frac{2}{15}$.

Divide using an equivalent fraction, where the numerator is a multiple of the whole number.

For example, $\frac{2}{5} \div 3 = \frac{6}{15} \div 3$

$= \frac{2}{15}$

2.9 Dividing Fractions Using a Related Multiplication

GOAL

Divide fractions using a related multiplication.

At-Home Help

To divide two fractions, you can:

Multiply by the reciprocal.
The reciprocal is the fraction that results from switching the numerator and denominator. For example, $\frac{4}{5}$ is the reciprocal of $\frac{5}{4}$.
Calculate using the reciprocal.

$$\frac{2}{3} \div \frac{4}{9} = \frac{2}{3} \times \frac{9}{4}$$
$$= \frac{18}{12}$$
$$= 1\frac{6}{12} \text{ or } 1\frac{1}{2}$$

1. Calculate.

a) $\frac{2}{3} \div \frac{3}{4} =$ ____ $\times$ ____ or ____

b) $\frac{4}{5} \div \frac{3}{5} =$ ____ $\times$ ____ or ____

c) $\frac{5}{6} \div \frac{1}{3} =$ ____ $\times$ ____ or ____

d) $\frac{7}{16} \div \frac{6}{20} =$ ____ $\times$ ____ or ____

2. Write =, <, or > in each box. Calculate the quotients that are greater than 2.

a) $\frac{6}{10} \div \frac{1}{4}$ ☐ 2; $\frac{6}{10} \div \frac{1}{4} =$ ____

b) $\frac{1}{3} \div \frac{2}{1}$ ☐ 2; $\frac{1}{3} \div \frac{2}{1} =$ ____

c) $\frac{7}{8} \div \frac{2}{5}$ ☐ 2; $\frac{7}{8} \div \frac{2}{5} =$ ____

d) $\frac{2}{3} \div \frac{1}{2}$ ☐ 2; $\frac{2}{3} \div \frac{1}{2} =$ ____

3. Why does it make sense that $\frac{6}{7} \div \frac{2}{3}$ is greater than $\frac{6}{7}$?

____

4. Julio delivers newspapers as his part-time job. It takes $5\frac{1}{2}$ min to deliver one newspaper.

a) How many newspapers can Julio deliver in 20 min? ____ $\div 5\frac{1}{2} =$ ______________

b) How many newspapers can Julio deliver in 45 min? ____ $\div 5\frac{1}{2} =$ ______________

c) How many newspapers can Julio deliver in $1\frac{1}{2}$ h? ____ $\div 5\frac{1}{2} =$ ______________

2.10 Order of Operations

GOAL

Use the order of operations in calculations involving fractions.

1. Calculate using the rules for order of operations.

a) $\left(\frac{1}{2} + \frac{1}{3}\right) \times \frac{6}{7} = \underline{\frac{5}{6}} \times \frac{6}{7}$

$\frac{3}{6} + \frac{2}{6}$ $= \underline{\frac{30}{42} \approx \frac{5}{7}}$

b) $\frac{3}{4} \div \left(\frac{1}{2} + \frac{1}{4}\right) = \frac{3}{4} \div \underline{\frac{3}{4}}$

$\frac{2}{4}$ $= \underline{1}$

c) $\frac{4}{6} \div \left(\frac{5}{7} \div \frac{1}{2}\right) + \frac{2}{4} = \frac{4}{6} \div \underline{\frac{10}{7}} + \frac{2 \div 2}{4 \div 2}$ $\frac{1 \times 30}{2 \times 30}$

$\frac{5}{7} \times \frac{2}{1}$ $= \underline{\frac{28}{60} + \frac{30}{60} = \frac{58 \div 2}{60 \div 2}} =$

$\frac{4}{6} \times \frac{7}{10} = \frac{28}{60}$ $= \frac{29}{30}$

d) $\frac{4}{6} \div \left(\frac{5}{7} \times \frac{1}{2} + \frac{2}{4}\right) = \frac{4}{6} \div \left(\underline{\frac{24}{28}}\right)$ → $\frac{6}{7}$

$\frac{5 \times 2}{14 \times 2} + \frac{2 \times 7}{4 \times 7}$ $= \frac{4}{6} \div \underline{\frac{6}{7}}$

$\frac{10}{28} + \frac{14}{28}$ $= \underline{\frac{4}{6} \times \frac{7}{6} = \frac{28 \div 4}{36 \div 4} \Rightarrow \frac{7}{9}}$

e) $\left(\frac{5}{10} - \frac{1}{3} \times \frac{2}{8} + \frac{1}{5}\right) \div \frac{1}{6} = \left(\frac{5}{10} - \frac{2}{24} + \frac{1}{5}\right) \div \underline{\frac{1}{6}}$

$\frac{5 \times 24}{10 \times 24} - \frac{2}{24} + \frac{1}{5}$ $= \underline{\frac{74}{120} \times \frac{6}{1} = \frac{444}{120}} \approx \frac{111}{30} \approx 3\frac{21}{30}$

$\frac{120}{240} - \frac{20}{240}$ $\frac{10}{24} + \frac{1}{5} = 3\frac{21}{30}$

$\frac{50}{120} + \frac{24}{120} = \frac{74}{120}$

2. Place brackets to make this equation true.

$3 \times \left(\frac{2}{3} + \frac{1}{3}\right) \div \frac{1}{4} = 12$

At-Home Help

The order of operations is:

- Perform the operations in the brackets first.
- Divide and multiply from left to right.
- Add and subtract from left to right.
- Write the answer as a mixed number.

For example:

$\frac{3}{2} - \frac{2}{5} \div \frac{1}{5} \times \frac{3}{10} + \frac{2}{3}$ Divide.

$= \frac{3}{2} - 2 \times \frac{3}{10} + \frac{2}{3}$ Multiply.

$= \frac{3}{2} - \frac{6}{10} + \frac{2}{3}$ Subtract.

$= \frac{15}{10} - \frac{6}{10} + \frac{2}{3}$

$= \frac{9}{10} + \frac{2}{3}$ Add.

$= \frac{27}{30} + \frac{20}{30}$

$= \frac{47}{30}$ or $1\frac{17}{30}$

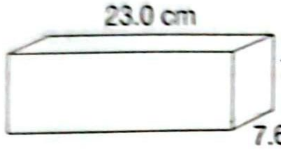
5.3 Determining the Surface Area of Prisms

GOAL

Develop strategies to calculate the surface area of prisms.

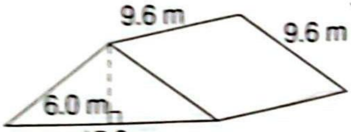
1. Calculate the surface area of each prism.

a)



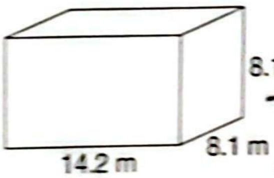
SA = 2(23 × 7.6) + 2(7.6 × 7.6) + 2(7.6 × 23)
= 814.72 cm²

b)



SA = 2(½(15)(6)) + 2(9.6 × 9.6) + (9.6 × 15)
= 418.32 cm²

c)



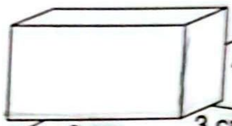
2(8.1 × 8.1) = 131.22
+ (14.2 × 8.1) × 2 = 230.04
+ 2(14.2 × 8.1) = 230.04
= 591.3 cm²

d)

At-Home Help

To determine the surface area of a prism:

- Calculate the area of each side.
- Add the areas.



$SA = 2 \times (8\text{ cm} \times 4\text{ cm})$
$+ 2 \times (8\text{ cm} \times 3\text{ cm})$
$+ 2 \times (3\text{ cm} \times 4\text{ cm})$
$= 64\text{ cm}^2 + 48\text{ cm}^2 + 24\text{ cm}^2$
$= 136\text{ cm}^2$

$SA = 2 \times (5.0\text{ cm} \times 6.0\text{ cm})$
$+ 6.0\text{ cm} \times 12.0\text{ cm}$
$+ 2 \times (\frac{1}{2} \times 6.0\text{ cm} \times 4.0\text{ cm})$
$= 60.0\text{ cm}^2 + 72.0\text{ cm}^2$
$+ 24.0\text{ cm}^2$
$= 156.0\text{ cm}^2$